RESEARCH REPORT

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GUGGENHEIM AERONAUTICAL LABORATORY

CALIFORNIA INSTITUTE OF TECHNOLOGY

HYPERSONIC RESEARCH PROJECT

Memorandum No. 56

July 15, 1960

CYLINDRICAL COUETTE FLOW IN A RAREFIED GAS ACCORDING TO GRAD'S EQUATIONS

by

Daniel Kwoh-i Ai

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CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

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ABSTRACT

Grad's thirteen moment method is applied to the problem of the shear flow and heat conduction between two concentric, rotating cylinders of infinite length. In order to concentrate on the effects of curvature the problem is linearized by requiring that the Mach number is small compared with unity, and that the temperature difference between the two cylinders is small compared with the mean temperature. The solutions of the linearized Grad equations show a qualitatively correct transition of the cylinder drag from free-molecule flow to the classical Navier-Stokes regime. However the magnitude of the curvature effect on the drag in rarefied flow is not given correctly, because Grad's distribution function ignores the wedge-like domains of influence of the two cylinders.

The solution obtained for the heat transfer rate is physically unrealistic in the free-molecule flow limit, and this result is produced by a cross-coupling between the normal stresses and the radial heat flux imposed by Grad's distribution function. In this simple problem the difficulty can be eliminated by taking the normal stresses to be identically zero and employing a truncated moment method. However,in general this device cannot be utilized in problems involving curved solid boundaries, or when dissipation is considered. One concludes that the choice of the distribution function to be employed in Maxwell's moment equations is dictated by the requirements imposed in the limiting case of highly rarefied gas flows, as well as in the Navier-Stokes regime.

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LIST OF SYMBOLS

a	radius of inner cylinder (rotating)
b	radius of outer cylinder (stationary)
$\vec{c}$	intrinsic velocity, $\vec{c} = \vec{\xi} - \vec{u}(x, t)$
C_D	drag coefficient
C_H	Stanton number
f	velocity distribution function
f_o	Maxwellian velocity distribution function $= \frac{\rho}{(2\pi RT)^{3/2}} \exp\left\{- c^2/2RT\right\}$
k	coefficient of heat conductivity $= 15/4\, R\mu_1$
p	pressure of gas
p_{ij}	stress (increment over hydrostatic pressure)
Pr	Prandtl number $= \frac{c_p \mu_1}{k}$
q_i	heat flux
r, θ	cylindrical coordinates
R	gas constant
Re	Reynolds number $= \rho_1 U \left(\frac{b-a}{\mu_1} \right)$
T	temperature of gas
u, v	velocity components in x, y direction of Cartesian coordinates
u_r, u_θ	velocity components in r, θ directions of cylindrical coordinates
U	velocity at surface of rotating wall
x, y	rectangular Cartesian coordinates
α	fraction of incident gas molecules specularly reflected from Cylinder walls
λ	mean free path of the gas molecules
μ	coefficient of viscosity
$\vec{\xi}$	molecule velocity

ρ density of gas

Subscripts

1 quantities in the gas at the surface of the inner cylinder

2 quantities in the gas at the surface of the outer cylinder

I. INTRODUCTION

Because of the well-known difficulties encountered in attempting to solve the Maxwell-Boltzmann integro-differential equation, a number of investigators have turned instead to Maxwell's integral equations of transfer.* In this procedure the Maxwell-Boltzmann equation is satisfied in a certain average sense, rather than point-by-point, and the particle velocity distribution function is regarded as a suitable weighting function. The first modern application of Maxwell's technique to fluid mechanics is H. Grad's[2] thirteen-moment method, which utilizes the "local Maxwellian", multiplied by a polynomial of the Chapman-Enskog type. The coefficients of this polynomial contain the corresponding stresses and heat flux quantities, which are now regarded as new dependent variables to be determined by solving thirteen simultaneous first-order differential equations obtained from the Maxwell moment equations. Of course in special problems the number of moments required is much less than thirteen.

Yang[3] and Lees applied Grad's method to the problem of the steady shearing motion and heat conduction between two infinite, parallel flat plates. In order to bring out some of the main features of Grad's method, without becoming involved in undue mathematical complications, the problem is linearized by requiring that the Mach number is small compared with unity, and that the temperature difference between the two plates is small compared with ambient temperature. Reasonable results

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* See Reference 1 for a brief review and discussion of some of this work.

for drag, heat transfer and velocity and temperature profiles were obtained over the whole range of gas densities. In the limit $Re/M \longrightarrow 0$ these results agree with the usual free-molecule flow quantities, while in the opposite limiting case $Re/M >> 1$ they join smoothly to the classical Navier-Stokes and Fourier relations.

Linearized, steady, plane Couette flow is undoubtedly too simple to provide a meaningful test of any integral method. One would like to investigate the influence of dissipation and streamline curvature on molecular effects. Such a study utilizing Grad's equations might be helpful in answering questions about the sensitivity of the results obtained by Maxwell's integral method to the form of the weighting function employed. One of the simplest situations involving curvature is the problem of shear flow and heat conduction between two concentric, rotating cylinders of infinite length (cylindrical Couette flow). In addition, this flow is one of the few that have been studied experimentally over the whole range of gas density by several different investigators[4, 5].

On the theoretical side, Rose[6] was the first to apply Grad's equations to cylindrical Couette flow, but the results were never published. In a private communication Dr. Grad states that no explicit solutions of the non-linear problem were obtained. Chiang[7] also had some difficulties with the non-linear Grad equations for this problem, and he resorted instead to an expansion procedure in powers of M^2/Re. Up to second-order terms this procedure is identical to the Burnett expansion[8], and is not very helpful for rarefied flows. In the present study the problem is linearized by requiring that $M^2 << 1$ and $\Delta T/T << 1$, in order to concentrate on the curvature effect. In Section II the full Grad equations and boundary conditions for steady, cylindrical Couette flow are written down, and the usual conservation integrals are obtained. In Section III the linearized equations and boundary conditions are formulated and solved, and the results compared with experiments and with the expression for the cylinder drag suggested in Reference 3. Section IV contains a critical discussion of the results, some conclusions about the difficulties inherent in utilizing Grad's form of the weighting function, and some observations on the rules that must guide the selection of a suitable weighting function.

II. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

II. 1. Equations of Motion for Cylindrical Couette Flow

Grad's general equations of motion for a two-dimensional problem in cylindrical coordinates are given in Appendix I. In the case of steady cylindrical Couette flow symmetry requires that all mean quantities are functions of r only; hence $(\partial/\partial t) = (\partial/\partial \theta) = 0$, and we are dealing with ordinary differential equations. The mean quantities involved in the problem are the following:

u_r , u_θ	r and θ components of the velocity vector
q_r , q_θ	r and θ components of the energy flux
p_{rr}, $p_{\theta\theta}$, $p_{r\theta}$	stress components (increment over hydrostatic pressure)
p, ρ, T	thermodynamic variables; pressure, density, and temperature .

Hence, we have ten unknowns to determine.

Grad's thirteen moment approximations furnish a set of nine moment equations for the 2-dimensional case; thus, one more equation is needed. This additional equation is obtained by relating the temperature to a certain second moment of the distribution function f. Since Grad's scheme is set up for monatomic molecules, each of which has three degrees of freedom, the kinetic energy per unit mass is 3/2 RT. An element of kinetic energy is $\frac{1}{2} c^2 f \, d\vec{x} \, d\vec{\xi}$; integration over all values of $\vec{\xi}$ yields the equation of state

$$3/2 \, RT = 1/\rho \int \tfrac{1}{2} c^2 f \, d\vec{\xi} = (3/2)(p/\rho)$$

or

$$p = \rho R T \tag{1}$$

The nine moment equations are

Continuity

$$(d/dr)(\rho\, u_r\, r) = 0 \tag{2}$$

Momentum

$$\frac{dp}{dr} + \frac{dp_{rr}}{dr} + \frac{p_{rr} - p_{\theta\theta}}{r} + \rho u_r \frac{du_r}{dr} - \rho \frac{u_\theta^2}{r} = 0 \tag{3}$$

$$\frac{dp_{r\theta}}{dr} + \frac{2 p_{r\theta}}{r} + \rho u_r \frac{du_\theta}{dr} + \rho \frac{u_\theta u_r}{r} = 0 \tag{4}$$

Energy

$$\frac{5}{3} p \left(\frac{du_r}{dr} + \frac{u_r}{r} \right) + \frac{2}{3} \left(\frac{dq_r}{dr} + \frac{q_r}{r} \right) + \frac{2}{3} \left(p_{rr} \frac{du_r}{dr} + p_{r\theta} \frac{du_\theta}{dr} + p_{\theta\theta} \frac{u_r}{r} - p_{r\theta} \frac{u_\theta}{r} \right) + u_r \frac{dp}{dr} = 0 \tag{5}$$

Stresses

$$\frac{2}{3} p \left(2 \frac{du_r}{dr} - \frac{u_r}{r} \right) + \frac{4}{15} \left(2 \frac{dq_r}{dr} - \frac{q_r}{r} \right) + u_r \frac{dp_{rr}}{dr} - \frac{2 u_\theta}{r} p_{r\theta} + \frac{7}{3} p_{rr} \frac{du_r}{dr} - \frac{2}{3} p_{r\theta} \frac{du_\theta}{dr} - \frac{4}{3} \frac{p_{r\theta}}{r} u_\theta - \frac{2}{3} \frac{p_{\theta\theta} u_r}{r} + \frac{p_{rr} u_r}{r} = - \frac{p}{\mu} p_{rr} \tag{6}$$

$$p\left(\frac{du_\theta}{dr} - \frac{u_\theta}{r}\right) + \frac{2}{5}\left(\frac{dq_\theta}{dr} - \frac{q_\theta}{r}\right) + u_r\frac{dp_{r\theta}}{dr} + \frac{u_\theta(p_{rr}-p_{\theta\theta})}{r}$$

$$+ p_{rr}\frac{du_\theta}{dr} + 2p_{r\theta}\frac{u_r}{r} + 2p_{r\theta}\frac{du_r}{dr} - p_{\theta\theta}\frac{u_\theta}{r} = -\frac{p}{\mu}p_{r\theta} \tag{7}$$

$$-\frac{2}{3}p\left(\frac{du_r}{dr} - 2\frac{u_r}{r}\right) - \frac{4}{15}\left(\frac{dq_r}{dr} - 2\frac{q_r}{r}\right) + \frac{2}{r}u_\theta p_{r\theta} + u_r\frac{dp_{\theta\theta}}{dr}$$

$$+\frac{4}{3}p_{r\theta}\frac{du_\theta}{dr} - \frac{2}{3}\left(p_{rr}\frac{du_r}{dr} - p_{r\theta}\frac{u_\theta}{r}\right) + \frac{7}{3}p_{\theta\theta}\frac{u_r}{r} + p_{\theta\theta}\frac{du_r}{dr} = -\frac{p}{\mu}p_{\theta\theta} \tag{8}$$

Heat Flux

$$\frac{5}{2}p\frac{dRT}{dr} + RT\left(\frac{dp_{rr}}{dr} + \frac{p_{rr}-p_{\theta\theta}}{r}\right) + q_r\left(\frac{du_r}{dr} + \frac{u_r}{r}\right) + u_r\frac{dq_r}{dr} - u_\theta\frac{q_\theta}{r}$$

$$+\frac{11}{5}q_r\frac{du_r}{dr} + \frac{2}{5}\left(q_r\frac{u_r}{r} + q_\theta\frac{du_\theta}{dr}\right) - \frac{7}{5}q_\theta\frac{u_\theta}{r} - \frac{p_{rr}}{\rho}\frac{dp}{dr} + \frac{7}{2}p_{rr}\frac{dRT}{dr}$$

$$-\frac{p_{rr}}{\rho}\left(\frac{dp_{rr}}{dr} + \frac{p_{rr}-p_{\theta\theta}}{r}\right) - \frac{p_{r\theta}}{\rho}\left(\frac{dp_{r\theta}}{dr} + \frac{2p_{r\theta}}{r}\right) = -\frac{2}{3}\frac{p}{\mu}q_r \tag{9}$$

$$RT\left(\frac{dp_{r\theta}}{dr} + \frac{2p_{r\theta}}{r}\right) + q_\theta\left(\frac{du_r}{dr} + \frac{u_r}{r}\right) + u_r\frac{dq_\theta}{dr} + q_r\frac{u_\theta}{r} + \frac{11}{5}q_\theta\frac{u_r}{r}$$

$$+\frac{7}{5}q_r\frac{du_\theta}{dr} + \frac{2}{5}\left(q_\theta\frac{du_r}{dr} - q_r\frac{u_\theta}{r}\right) - \frac{p_{r\theta}}{\rho}\frac{dp}{dr} + \frac{7}{2}p_{r\theta}\frac{dRT}{dr} \tag{10}$$

$$-\frac{p_{r\theta}}{\rho}\left(\frac{dp_{rr}}{dr} + \frac{p_{rr}-p_{\theta\theta}}{r}\right) - \frac{p_{\theta\theta}}{\rho}\left(\frac{dp_{r\theta}}{dr} + \frac{2p_{r\theta}}{r}\right) = -\frac{2}{3}\frac{p}{\mu}q_\theta$$

The right-hand side terms in stresses and heat flux equations are produced by the collision integral. In the heat flux equations, the results of the stress equations are already utilized; hence $-(2/3 \;\; p/\mu)\, q_i$ are the only terms introduced by the collision integral.

II. 2. Boundary Conditions

Since we are dealing with a cylindrical coordinate system which is orthogonal, we have a local Cartesian coordinate system; hence the boundary conditions for plane Couette flow can be applied to the cylindrical case if we simply replace the subscript x by θ and y by r.

At the outer stationary cylinder (r = b) the boundary conditions are as follows[3]:

$$u_r(b) = 0 \tag{11}$$

$$-\frac{p_{r\theta}(b)}{p(b)} + \frac{2(1-\alpha)}{(1+\alpha)}\frac{u_\theta(b)}{[2\pi RT(b)]^{\frac{1}{2}}}\left(1+\frac{p_{rr}(b)}{2p(b)}\right) + \frac{2}{5}\frac{(1-\alpha)}{(1+\alpha)}\frac{q_\theta(b)}{[2\pi RT(b)]^{\frac{1}{2}}p(b)} = 0 \tag{12}$$

$$-\left[\frac{2\pi}{RT(b)}\right]^{\frac{1}{2}}\frac{q_r(b)}{p(b)} + \frac{4(1-\alpha)}{(1+\alpha)}\left[1-\frac{T_2}{T(b)} - \frac{p_{rr}(b)}{2p(b)}\left(\frac{3}{2}-\frac{T_2}{T(b)}\right)+\left(1+\frac{p_{rr}(b)}{2p(b)}\right)\frac{\frac{[u_\theta(b)]^2}{2}}{RT(b)}\right] = 0 \tag{13}$$

At the inner rotating cylinder (r = a)

$$u_r(a) = 0 \tag{14}$$

$$\frac{p_{r\theta}(a)}{p(a)} + \frac{2(1-\alpha)}{(1+\alpha)}\frac{u_\theta(a)-U}{[2\pi RT(a)]^{\frac{1}{2}}}\left(1+\frac{p_{rr}(a)}{2p(a)}\right) + \frac{2}{5}\frac{(1-\alpha)}{(1+\alpha)}\frac{q_\theta(a)}{[2\pi RT(a)]^{\frac{1}{2}}p(a)} = 0 \tag{15}$$

$$\left[\frac{2\pi}{RT(a)}\right]^{\frac{1}{2}}\frac{q_r(a)}{p(a)} + \frac{4(1-\alpha)}{(1+\alpha)}\left[1-\frac{T_1}{T(a)} + \frac{p_{rr}(a)}{2p(a)}\left(\frac{3}{2}-\frac{T_1}{T(a)}\right)+\left(1+\frac{p_{rr}(a)}{2p(a)}\right)\frac{\frac{(u_\theta(a)-U)^2}{2}}{RT(a)}\right] = 0 \tag{16}$$

Equations (11) and (14) represent conservation of mass. Equations (12), (15) and (13), (16) represent conservation of momentum and energy, respectively, at outer and inner walls.

II. 3. General Solutions of Cylindrical Flow

From the integration of Eq. (2) and with the aid of the boundary conditions (11) and (14), we obtain that everywhere

$$u_r = 0 \quad . \tag{17}$$

This result shows clearly that only shear flow exists. By utilizing this result, the rest of the differential equations are simplified enormously and we are able to integrate the conservation equations immediately [Eqs. (4) and (5)]:

$$p_{r\theta} = B/r^2 \tag{18}$$

$$r(q_r + p_{r\theta}\, u_\theta) = c \quad . \tag{19}$$

Equation (18) states that the torque is constant across the annulus, while Eq. (19) states that the flux of heat energy plus the rate at which work is done on the fluid by the shear stress is a constant. So far the integrals obtained are valid for the flow between two concentric cylinders at arbitrary temperature difference and rotating speed. A study of the equations shows that solutions in reasonably simple form are difficult to find. In order to bring out the effect of curvature as simply as possible linearized equations and boundary conditions will be used instead.

III. CYLINDRICAL COUETTE FLOW AT LOW MACH NUMBERS AND SMALL TEMPERATURE DIFFERENCES

III. 1. The Linearization of Equations of Motion and Boundary Conditions

When the inner cylinder is rotating slowly and the temperature difference between the cylinders is kept small, or more precisely if

$$M = U/\sqrt{\gamma R T_1} \quad << 1$$

and

$$\frac{T_1 - T_2}{T_1} >> 1 \quad ,$$

the thermodynamical quantities may be expressed as

$$\begin{aligned} \rho &= \rho_1 + \rho' \\ p &= p_1 + p' \\ T &= T_1 + T' \end{aligned} \tag{20}$$

and the coefficient of viscosity

$$\mu = \mu_1 + \mu' \quad . \tag{21}$$

For the remaining quantities, we have

$$Q = Q' \quad , \tag{22}$$

where Q denotes any velocity component, stress, or heat flux component. Subscript 1 denotes quantities evaluated at the inner cylinder, used here as the reference base, and the prime denotes perturbations.

If the expressions (20) to (22) are introduced into Eqs. (1) to (16) and all quadratic terms of small perturbations are neglected, the equations of motion as well as the boundary conditions are linearized. Furthermore, the tangential quantities u_θ, $p_{r\theta}$, q_θ and the normal quantities ρ, p (or T), p_{rr}, $p_{\theta\theta}$, and q_r are separated. This uncoupling of tangential and normal quantities has been pointed out by Yang and Lees[3, 9] as being typical of the particular linearization procedure. The remaining equations of motion in linearized form are as follows:

Momentum

$$\frac{dp'}{dr} + \frac{dp'_{rr}}{dr} + \frac{p'_{rr} - p'_{\theta\theta}}{r} = 0 \tag{23}$$

Stresses

$$\frac{4}{15}\left(2\,\frac{dq'_r}{dr} - \frac{q'_r}{r}\right) = -\frac{p_1}{\mu_1}\,p'_{rr} \tag{24}$$

$$p_1\left(\frac{du'_\theta}{dr} - \frac{u'_\theta}{r}\right) + \frac{2}{5}\left(\frac{dq'_\theta}{dr} - \frac{q'_\theta}{r}\right) = -\frac{p_1}{\mu_1}\,p'_{r\theta} \tag{25}$$

$$-\frac{4}{15}\left(\frac{dq'_r}{dr} - 2\,\frac{q'_r}{r}\right) = -\frac{p_1}{\mu_1}\,p'_{\theta\theta} \tag{26}$$

Heat Flux

$$\frac{5}{2}p_1\,\frac{dRT'}{dr} - RT_1\,\frac{dp'}{dr} = -\frac{2}{3}\frac{p_1}{\mu_1}\,q_r' \tag{27}$$

$$q_\theta' = 0 \tag{28}$$

State

$$p' = \rho' RT_1 + \rho_1 RT' \tag{29}$$

After linearization the shear stress $p_{r\theta}$ is given by a form identical to the Navier-Stokes stress strain rate relation [Eqs. (25) and (28)]; therefore, we obtain the same expression for u_θ as in the incompressible Navier-Stokes solution. However, the normal stresses p_{rr} and $p_{\theta\theta}$ are described by more complicated expressions. Here the heat flux rates are coupled in with the normal stresses. This coupling is inherent in Grad's scheme and as a result introduces difficulties in the heat transfer problem. (See Sections III. 4 and IV.)

As far as Eq. (27) is concerned, we obtain the familiar Fourier conduction law after simply identifying the coefficient of heat conductivity k by $15/4\, R\mu_1$.

The linearized boundary conditions are at r = b,

$$-\frac{p'_{r\theta}(b)}{p_1} + \frac{2(1-\alpha)}{(1+\alpha)} \frac{u'_\theta(b)}{[2\pi R T_1]^{\frac{1}{2}}} = 0 \tag{30}$$

$$-\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}} \frac{q'_r(b)}{p_1} + \frac{4(1-\alpha)}{(1+\alpha)}\left[\frac{T_1-T_2}{T_1} + \frac{T_2}{T_1}\frac{T'(b)}{T_1} - \frac{p'_{rr}(b)}{4p_1}\right] = 0 \tag{31}$$

and at r = a,

$$\frac{p'_{r\theta}(a)}{p_1} + \frac{2(1-\alpha)}{(1+\alpha)} \frac{u'_\theta(a) - U}{[2\pi R T_1]^{\frac{1}{2}}} = 0 \tag{32}$$

$$\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{q_r'(a)}{p_1}+\frac{4(1-\alpha)}{(1+\alpha)}\left[\frac{T'(a)}{T_1}+\frac{p_{rr}'(a)}{4p_1}\right]=0 \tag{33}$$

III. 2. The Solution of the Linearized Equations of Motion

By simplifying Eqs. (5) and (19) we have

$$q_r' = c/r \quad . \tag{34}$$

The integration of Eq. (25) yeilds

$$u_\theta' = c_1 r + \frac{B}{2\mu_1 r} \quad , \tag{35}$$

which is the same expression as the incompressible Navier Stokes solution.

Eqs. (24) and (26) give the normal stresses

$$p_{rr}' = \frac{\mu_1}{p_1}\frac{4c}{5r^2} \tag{36}$$

$$p_{\theta\theta}' = -\frac{\mu_1}{p_1}\frac{4c}{5r^2} \tag{37}$$

These normal stresses are identically zero in the Navier Stokes solution because $\mathrm{div}\ \vec{u} = 0$, $u_r = 0$, and $u_\theta = u_\theta(r)$.

Substituting p_{rr}' and $p_{\theta\theta}'$ into Eq. (23) we arrive at

$$dp'/dr = 0 \qquad \text{or} \qquad p' = \text{constant} \tag{38}$$

Since the pressure is defined as $p = p_1 + p'$, we see that p' is just an additive constant; therefore, it may be set equal to zero, i. e., $p = p_1$ throughout the flow field. Knowing q_r' we can integrate Eq. (27) to give

$$T' = -(4/15)\,\frac{c}{\mu_1 R}\,\ln r + c_2 \quad . \tag{39}$$

T' satisfies the Laplace equation and it is natural we obtain the term $\ln r$ in cylindrical coordinates. Finally, with the aid of the equation of state we have

$$p' = (4/15)\,\frac{c}{\mu_1 R}\,\frac{\rho_1}{T_1}\,\ln r - (\rho_1/T_1)\,c_2 \quad . \tag{40}$$

III. 3. The Evaluation of Tangential Quantities -- Shear Stress, Tangential Velocity, and Heat Flux

The tangential quantities $p_{r\theta}$, u_θ ($q_\theta = 0$) given by the linearized Grad equations are identical in form with the Navier-Stokes solution, but the boundary conditions are quite different. We shall express these quantities in such a way that the quantity Re/M appears as a parameter. The constants B and c_1 in $p_{r\theta}$ and u_θ may be evaluated by utilizing the boundary conditions (30) and (32). After substituting the results of Eqs. (18), (35), and (38), Eqs. (30) and (32) become

$$-\frac{B}{p_1 b^2} + 2\,\frac{(1-\alpha)}{(1+\alpha)}\,\frac{1}{\sqrt{2\pi R T_1}}\left(c_1 b + \frac{B}{2\mu_1 b}\right) = 0 \tag{41}$$

$$\frac{B}{p_1 a^2} + \frac{2(1-\alpha)}{(1+\alpha)}\,\frac{1}{\sqrt{2\pi R T_1}}\left(c_1 a + \frac{B}{2\mu_1 a}\right) = \frac{2(1-\alpha)}{(1+\alpha)}\,\frac{U}{\sqrt{2\pi R T_1}} \,. \tag{42}$$

By solving these simultaneous equations for B and c_1 , we obtain

$$B = \frac{\frac{2b^2}{b-a} U \mu_1 Re/M}{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\left(\frac{a}{b}+\frac{b^2}{a^2}\right)+\frac{a+b}{a}\frac{Re}{M}} \tag{43}$$

$$c_1 = \frac{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}} - \frac{Re}{M}\frac{b}{b-a}}{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\left(\frac{a}{b}+\frac{b^2}{a^2}\right)+\frac{a+b}{a}\frac{Re}{M}} \frac{U}{b} \tag{44}$$

where

$$M = \frac{U}{\sqrt{\frac{5}{3} RT_1}} \qquad \text{and} \qquad Re = \rho_1 U \frac{(b-a)}{\mu_1} \quad ,$$

for a monatomic gas.

Hence, we obtain

$$p_{r\theta} = \frac{\frac{2b^2}{b-a} U \mu_1 \frac{Re}{M}}{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\left(\frac{a}{b}+\frac{b^2}{a^2}\right)+\frac{a+b}{a}\frac{Re}{M}} \frac{1}{r^2} \tag{45}$$

$$u_\theta/U = \frac{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\frac{r}{b} + \frac{Re}{M}\frac{b}{b-a}\left(\frac{b}{r}-\frac{r}{b}\right)}{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\left(\frac{a}{b}+\frac{b^2}{a^2}\right)+\frac{a+b}{a}\frac{Re}{M}} \tag{46}$$

The velocity profile u_θ/U across the annulus is plotted in Figure 2 for different values of Re/M.

The drag coefficient C_D multiplied by the Mach number M is defined as

$$C_D M = \frac{p_{r\theta} M}{\frac{1}{2} \rho_1 U^2} \quad .$$

At the stationary wall (r = b), $p_{r\theta}(b) = B/b^2$; hence

$$C_D M = \frac{4}{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\left(\frac{a}{b}+\frac{b^2}{a^2}\right) + \frac{a+b}{a}\frac{Re}{M}} \tag{47}$$

or

$$\frac{1}{C_D M} = \frac{1}{4} \frac{1+\alpha}{1-\alpha} \left(\frac{a}{b} + \frac{b^2}{a^2} \right) \sqrt{\frac{10\pi}{3}} + \frac{1}{2} \left(1 + \frac{b-a}{2a} \right) \frac{Re}{M} \quad . \tag{48}$$

For a diffusively reflecting surface ($\alpha = 0$) ,

$$\frac{1}{C_D M} = \frac{1}{4} \left(\frac{a}{b} + \frac{b^2}{a^2} \right) \sqrt{\frac{10\pi}{3}} + \frac{1}{2} \left(1 + \frac{b-a}{2a} \right) \frac{Re}{M} \quad . \tag{49}$$

The above expression is in complete agreement with the result obtained by C. Y. Liu for small ratio of annulus width to cylinder radius in his analysis based on L. Lees method.[1] In the limiting case when a approaches b, $1/C_D M$ takes the form of Eq. (50), which is identical to the result for plane Couette flow found by Yang[3] and Lees.

$$1/C_D M = \frac{1}{2} \left(\sqrt{\frac{10\pi}{3}} + \frac{Re}{M} \right) \tag{50}$$

In that paper it was suggested that the drag on the stationary outer cylinder can be written in the form

$$1/C_D M = A(b/a) + B(b/a)\ Re/M \quad .$$

By analogy with the case of plane Couette flow it was thought that the function B(b/a) should be identical with the expression for drag obtained in the Navier-Stokes regime, and Eq. (49) shows that this supposition is correct. However, the function A(b/a) was taken from the free molecule flow result of Bowyer[4] and Talbot, i.e.,

$$A(b/a) = \tfrac{1}{2}(b^2/a^2)\ \sqrt{10\pi/3} \quad .$$

Evidently the value of the drag coefficient given by Eq. (49) in the limit $Re/M \longrightarrow 0$ is $2(1 + a^3/b^3)^{-1}$ times larger than the correct value. (See Section IV.)

To determine the slip velocity, we have at r = b

$$\frac{u_\theta(b)}{U} = \frac{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}}{\frac{1+\alpha}{1-\alpha}\sqrt{\frac{10\pi}{3}}\left(\frac{a}{b}+\frac{b^2}{a^2}\right)+\frac{a+b}{a}\frac{Re}{M}} \tag{51}$$

and at r = a

$$\frac{u_\theta(a)}{U} = \frac{u_\theta(b)}{U} + \frac{\frac{a+b}{a}\frac{Re}{M}}{\frac{1+\alpha}{1-\alpha}\left(\frac{a}{b}+\frac{b^2}{a^2}\right)\sqrt{\frac{10\pi}{3}}+\frac{a+b}{a}\frac{Re}{M}} \quad . \tag{52}$$

In the limit when $Re/M \longrightarrow \infty$, we obtain

$$\frac{u_\theta(b)}{U} = 0 \tag{53}$$

$$\frac{u_\theta(a)}{U} = 1 \quad . \tag{54}$$

These results represent nothing but the usual no-slip boundary conditions associated with the "Navier-Stokes" limit.

In the other limit when $Re/M \longrightarrow 0$ we obtain

$$\frac{u_\theta(a)}{U} = \frac{u_\theta(b)}{U} = \frac{1}{(a/b) + (b^2/a^2)} \quad . \tag{55}$$

Furthermore, we have

$$\frac{u_\theta(b)}{U} = \frac{u_\theta(a)}{U} = \tfrac{1}{2} \tag{56}$$

if the gap b-a approaches zero, so that again the results are reduced to that of the plane case in the limit $b/a \longrightarrow 1$. The variations of $\frac{u_\theta(a)}{U}$ and $1/C_D M$ vs. Re/M for a diffusively reflecting surface ($\alpha = 0$) are plotted in Figures 3 and 4, respectively.

III. 4. The Evaluation of Normal Quantities -- Normal Stresses, Normal Heat Flux, and Thermodynamic Variables

As we pointed out earlier, the normal and tangential quantities are uncoupled after the linearization, so that what is left here reduces to the case of steady state heat conduction between two cylinders at rest. The whole problem will be solved after the evaluation of the two remaining constants c and c_2, and this can be done by substituting q_r', p_{rr}', and

T' into the two remaining boundary conditions Eqs. (31) and (33). We have therefore

$$\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{c}{bp_1} - \frac{4(1-\alpha)}{(1+\alpha)}\left[\frac{T_1-T_2}{T_1} - \frac{T_2}{T_1}\frac{4}{15}\frac{c}{\mu_1 RT_1}\ln b + \frac{T_2}{T_1}\frac{c_2}{T_1} - \frac{c\mu_1}{5p_1^2 b^2}\right] = 0 \tag{57}$$

$$\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{c}{ap_1} + \frac{4(1-\alpha)}{(1+\alpha)}\left[-\frac{4}{15}\frac{c}{\mu_1 RT_1}\ln a + \frac{c_2}{T_1} + \frac{1}{5}\frac{c\mu_1}{p_1^2 a^2}\right] = 0 \tag{58}$$

By solving these simultaneous equations for c and c_2, we obtain

$$c = \frac{\dfrac{T_1 - T_2}{T_1}}{\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{1}{bp_1}\left(1+\frac{b}{a}\frac{T_2}{T_1}\right)\frac{(1+\alpha)}{4(1-\alpha)} + \frac{T_2}{T_1}\frac{4}{15R\mu_1 T_1}\ln\frac{b}{a} + \frac{\mu_1}{5p_1^2 b^2}\left(1+\frac{b^2}{a^2}\frac{T_2}{T_1}\right)} \tag{59}$$

and

$$c_2 = -(T_1-T_2)\frac{\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{1}{ap_1}\frac{1+\alpha}{4(1-\alpha)} - \frac{4}{15R\mu_1 T_1}\ln a + \frac{\mu_1}{5p_1^2 a^2}}{\left(\frac{2\pi}{RT_1}\right)^{\frac{1}{2}}\frac{1}{bp_1}\left(1+\frac{b}{a}\frac{T_2}{T_1}\right)\frac{(1+\alpha)}{4(1-\alpha)} + \frac{T_2}{T_1}\frac{4}{15R\mu_1 T_1}\ln\frac{b}{a} + \frac{\mu_1}{5p_1^2 b^2}\left(1+\frac{b^2}{a^2}\frac{T_2}{T_1}\right)} \tag{60}$$

Hence the temperature variation across the annulus can be written as ($\alpha = 0$)

$$\frac{T_1 - T}{T_1 - T_2} = \frac{4}{15} \frac{\frac{Re}{M} \ln\left(1 + \frac{b-a}{a}\frac{r-a}{b-a}\right)}{\frac{1}{4}\sqrt{\frac{10\pi}{3}}\,\frac{b-a}{b}\left(1 + \frac{b}{a}\frac{T_2}{T_1}\right) + \frac{T_2}{T_1}\frac{Re}{M}\ln\frac{b}{a} + \frac{5}{4}\frac{M}{Re}\left(\frac{b-a}{b}\right)^2\left(1 + \frac{b^2}{a^2}\frac{T_2}{T_1}\right)}$$

$$+ \frac{\sqrt{\frac{10\pi}{3}}\,\frac{15}{16}\,\frac{b-a}{a} + \frac{5}{4}\frac{M}{Re}\left(\frac{b-a}{a}\right)^2}{\sqrt{\frac{10\pi}{3}}\,\frac{15}{16}\,\frac{b-a}{b}\left(1 + \frac{b}{a}\frac{T_2}{T_1}\right) + \frac{Re}{M}\frac{T_2}{T_1}\ln\frac{b}{a} + \frac{5}{4}\frac{M}{Re}\left(\frac{b-a}{b}\right)^2\left(1 + \frac{b^2}{a^2}\frac{T_2}{T_1}\right)} \tag{61}$$

The temperature profile, Eq. (61), across the annulus is plotted in Figure 5 for a diffusively reflected surface ($\alpha = 0$). The temperature of the gas at the surface of the inner cylinder is given by

$$T(a) = T_1 + \frac{b}{a}(T_2 - T_1)\frac{\sqrt{\frac{10\pi}{3}} + \frac{4}{3}\frac{1-\alpha}{1+\alpha}\frac{M}{Re}\frac{b-a}{a}}{\left[\sqrt{\frac{10\pi}{3}}\left(1 + \frac{b}{a}\frac{T_2}{T_1}\right) + \frac{T_2}{T_1}\frac{4(1-\alpha)4}{(1+\alpha)15}\frac{Re}{M}\frac{b}{b-a}\ln\frac{b}{a} + \frac{4}{3}\frac{(1-\alpha)}{(1+\alpha)}\frac{M}{Re}\frac{b-a}{b}\left(1 + \frac{b^2}{a^2}\frac{T_2}{T_1}\right)\right]} \tag{62}$$

In the limiting case when $Re/M \longrightarrow 0$,

$$T_1 - T(a) = (T_1 - T_2)\frac{1}{a^2/b^2 + \frac{T_2}{T_1}} \tag{61}$$

Furthermore, when a approaches b and T_2 approaches T_1 we obtain

$$T_1 - T(a) = \tfrac{1}{2}(T_1 - T_2) \tag{62}$$

In the other limiting case when $Re/M \longrightarrow \infty$

$$T_1 - T(a) = 0 \tag{63}$$

As expected there is no temperature jump at normal density. At the surface of the outer cylinder we have in the limiting case when $Re/M \longrightarrow 0$

$$T(b) - T_2 = (T_1 - T_2)\frac{1 - \frac{b^2}{a^2}\frac{T_1 - T_2}{T_1}}{1 + \frac{b^2}{a^2}\frac{T_2}{T_1}} \tag{64}$$

The ratio of the two temperature jumps is

$$\frac{T_1 - T(a)}{T(b) - T_2} = \frac{b^2/a^2}{1 + b^2/a^2\, T_2/T_1} \tag{65}$$

In the limit when a approaches b and T_2 approaches T_1, one recovers the result of plane Couette flow found by Yang[3] and Lees.

$$T_1 - T(a) = T(b) - T_2 = \tfrac{1}{2}(T_1 - T_2) \tag{66}$$

The products of Stanton number C_H and the Mach number M is defined as

$$C_H M = \frac{q_r\, M}{\rho_1\, U\, c_p\,(T_1 - T_2)}$$

At the wall of the inner cylinder we have

$$C_H M = \frac{1}{\frac{5}{2}\left[\frac{1+\alpha}{4(1-\alpha)}\frac{a}{b}\left(1+\frac{b}{a}\frac{T_2}{T_1}\right)\sqrt{\frac{10\pi}{3}} + \frac{T_2}{T_1}\frac{Re}{M}Pr\,\frac{a}{b-a}\ln\frac{b}{a} + \frac{M}{Re}\left(1+\frac{b^2}{a^2}\frac{T_2}{T_1}\right)\frac{5}{6}\frac{a}{b}\,\frac{b-a}{b}\right]} \tag{67}$$

or

$$\frac{1}{C_H M} = \frac{(1+\alpha)}{4(1-\alpha)}\frac{a}{b}\left(1+\frac{b^2}{a^2}\frac{T_2}{T_1}\right)\frac{5}{2}\sqrt{\frac{10\pi}{3}} + \frac{T_2}{T_1}\frac{a}{b-a}\ln\frac{b}{a}\,Pr\,\frac{Re}{M}$$
$$+\frac{5}{6}\frac{a^2}{b^2}\left(\frac{b-a}{a}\right)\frac{M}{Re}\left(1+\frac{b^2}{a^2}\frac{T_2}{T_1}\right) \tag{68}$$

One may notice that there are two kinds of limiting processes which lead to quite different results.

(1) Let a approach b at a fixed value of Re/M (also γ is taken to be 5/3 for a monatomic gas), then we again have the result of plane Couette flow[3].

$$1/C_H M = (5/4)\,\frac{(1+\alpha)}{(1-\alpha)}\sqrt{\frac{10\pi}{3}} + Pr\,(Re/M) \tag{69}$$

(2) For arbitrary a and b, we have as $Re/M \longrightarrow \infty$

$$1/C_H M \longrightarrow \frac{T_2}{T_1} \frac{a}{b-a} \ln \frac{b}{a} (Re/M) \; Pr \tag{70}$$

or

$$C_H \longrightarrow \frac{T_1}{T_2} \frac{b-a}{a} \frac{1}{\ln b/a} \frac{1}{Pr \; Re/M} \tag{71}$$

which agrees with the classical solution of Fourier heat conduction for steady state.

However, when $Re/M \longrightarrow 0$, we encounter the difficulty that $C_H M$ approaches zero instead of the constant value given by a free molecule calculation.

Thus, in a free molecular limit the temperature distribution and heat transfer are physically unrealistic. (See Section IV.) The plot of $C_H M$ vs. Re/M is given by Figure 6.

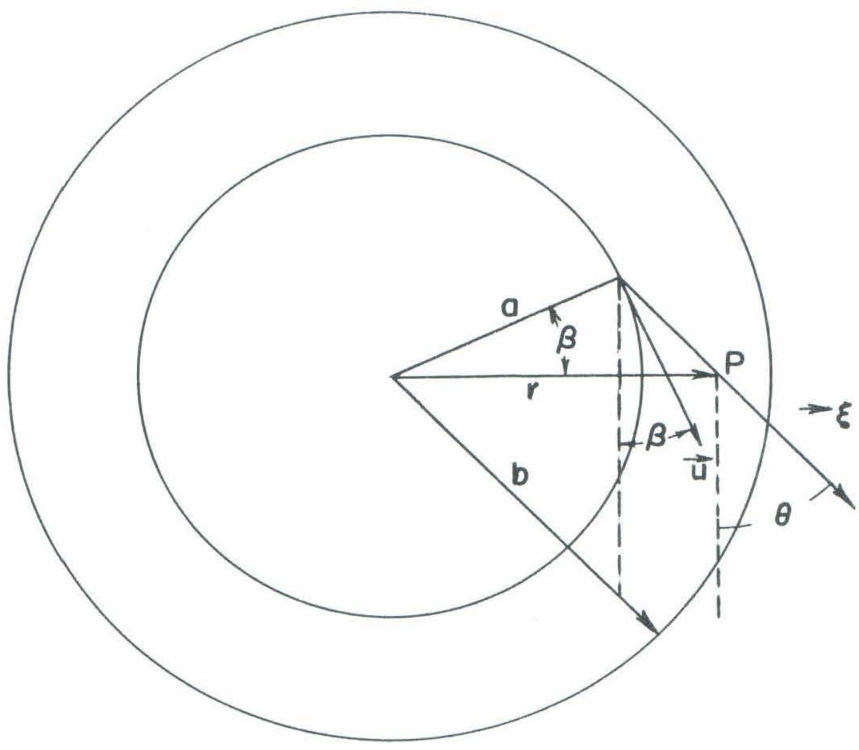
IV. DISCUSSION AND CONCLUSIONS

IV. 1. Cylinder Drag and Shear Flow

A number of writers[10, 1] have pointed out that Grad's distribution function is not expected to be entirely satisfactory for highly rarefied gas flows, because it does not contain the "two-sidedness", or discontinuity in velocity space that is so characteristic of the low density regime. In the present problem the actual distribution function in the limit $Re/M \longrightarrow 0$ for particles emitted diffusely from the inner rotating cylinder is given by the following expression (See sketch.):

$$f_P(\vec{\xi}, r) = n_a \left(\frac{m}{2\pi k T_a} \right)^{3/2} \exp\left\{ - \frac{m}{2kT_a} \left[\vec{\xi} - \vec{u}(\beta) \right]^2 \right\} \quad (72)$$

$$0 \leq \beta \leq \cos^{-1}(a/r) \quad ,$$



where $\vec{u}(\beta)$ is constant in magnitude, but not in direction, and β is uniquely determined by θ and (r/a)*. The velocity distribution function for particles emitted diffusely from the outer stationary cylinder is similar, except that $\vec{u} = 0$ and n_a, T_a are replaced by n_b, T_b. On the other hand, Grad's distribution function [Eq. (AI. 3)] ignores the wedge-like domains of influence of the two cylinders at the point P, as well as the angular dependence of $\vec{u}$ [Eq. (72)]. In the present problem these omissions lead inevitably to the Navier-Stokes relation for the shear stress $p_{r\theta}$ when $M^2 << 1$ [Eqs. (25) and (28)] **. The boundary conditions [Eqs. (30) and (32)] are reduced to the same form as Maxwell's famous velocity slip relation, thus assuring a qualitatively correct transition of the cylinder drag from free molecule flow to the classical Navier-Stokes regime [Eq. (48)].

When the width of the annulus is small compared to the inner cylinder radius the solutions of the linearized Grad equations for $p_{r\theta}$ and u_θ contain only small errors of order $(\frac{b-a}{a})$. But when $\frac{b-a}{a} = 0(1)$ the wedge-like domains of influence of the two cylinders cannot be ignored. For example, in the limit $Re/M \longrightarrow 0$ the effect of the inner cylinder rotation on the gas dies off with radial distance like the solid

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* In fact, $\tan\theta = \frac{(r/a) - \cos\beta}{\sin\beta}$, $\cos^{-1}(a/r) \leq \theta \leq \pi/2$.

** Actually this statement is applicable to any function of the form $f = f_o [1 + \phi]$ making the same omissions. We remark that a two-sided function utilizing half-range Maxwellians of the type introduced in Reference 1 yields the Navier-Stokes relation only when $\frac{b-a}{a} << 1$, but not otherwise.

angle subtended by the cylinder at any point in the annulus, and the linear mean velocity distribution given by Eq. (46) no longer represents the true physical situation. In particular, when $b/a >> 1$ the mean velocity given by Eq. (46) approaches zero everywhere, and $u_\theta(a) \longrightarrow 0$, instead of $\frac{1}{2}$. Thus the drag on the inner cylinder given by Eq. (45) is exactly twice the correct free-molecule flow value.

We conclude that the excellent agreement obtained between the solutions of the linearized Grad equations for steady, plane Couette flow[3] in the limit $Re/M \longrightarrow 0$ and the correct free-molecule quantities is somewhat misleading. This linearized plane flow problem is so simple that almost any reasonable distribution function employed in Maxwell's moment equations yields satisfactory results. The present study shows that the magnitude of the effect of streamline curvature on shear drag is not given correctly by Grad's f, but that at least there are no gross physical contradictions, so far as shear drag is concerned. The mean velocity distribution is less satisfactory. Similar conclusions can be drawn from a study of Goldberg's[11] solution of the linearized Grad equations for the "slow" flow over a sphere.

IV. 2. Heat Transfer and Mean Temperature Distribution

Even for small (but finite) values of $(b-a)/a$, the heat transfer rate given by the solution of the linearized Grad equations approaches zero faster than the density in the limit $Re/M \longrightarrow 0$ [Eq. (68)]. On the other hand, if $Re/M \sim (b-a)/\lambda$ is held fixed, while $(b-a)/a \longrightarrow 0$, we recover the results obtained previously[3] for linearized plane Couette flow, and the heat transfer rate approaches the correct free-molecule flow limit as $Re/M \rightarrow 0$. This non-uniform convergence and physically unrealistic behavior of the heat transfer solutions for linearized cylindrical Couette flow is produced by the coupling between the normal stress p_{rr} and the radial heat flux q_r, which occurs both in the moment equation for p_{rr} [Eq. (24)] and in the energy boundary condition at either cylinder [Eqs. (31) and (33)]. By dimensional analysis one can easily verify that a term of order M/Re is thereby introduced. The coupling between p_{rr} and q_r in the moment equation, in turn, is forced upon us by the term

$$- f_o \frac{q_r c_r}{pRT} \left(1 - \frac{c^2}{5RT} \right)$$

in Grad's distribution function. For example, on the left-hand side of Maxwell's moment equation for p_{rr} one obtains

$$(1/r)\,(d/dr) \left\{ rm \int f \, \xi_r^{\,3} \; d\vec{\xi} \right\},$$

and the term containing q_r in Grad's f evidently gives rise to a term of the form $(1/r)\,(d/dr)\,(rq_r)$ in this equation.

Similarly, the rate of energy transfer to the surface, given by the expression

$$m \int_{c_r < 0} f \;\; c_r \;\; (c^2/2) \;\; d\vec{\xi}$$

contains a term proportional to p_{rr}, because of the term $f_o\,(p_{rr}/2pRT)\,c_r^{\;2}$ appearing in Grad's distribution function. In linearized plane Couette flow $p_{yy} = p_{xx} \equiv 0$ and q_y = constant, so that no cross-coupling occurs. Such cross-couplings do not occur in the moment equation for the shear stress $p_{r\theta}$, because (1) the term containing $p_{r\theta}$ in Grad's f is anti-symmetric; (2) all physical quantities are functions of r alone in this particular problem; (3) $q_\theta \longrightarrow 0$ when $M^2 << 1$.

Because the geometry of the present problem is so simple this dilemma can be resolved by a slight modification of Grad's method. When $M^2 << 1$ the radial momentum equation shows that an acceptable solution is given by $p_{rr} = p_{\theta\theta} = 0$, $p =$ constant [Eqs. (3) and (23)]. But $q_{rr} \sim (1/r)$, so that this solution is clearly incompatible with the stress equations for p_{rr} and $p_{\theta\theta}$ [Eqs. (24) and (26)]. Therefore we must drop these two moment equations entirely, and agree to employ a modified Grad distribution function involving only $p_{r\theta}$, q_r , and q_θ . When this truncated moment method is utilized, the shear flow and cylinder drag are not changed, Eq. (27) for q_r is again reduced to the ordinary Fourier heat-conduction "law", and the energy boundary condition reduces to the well-known temperature-jump condition. Without going into details (Appendix 2), we state that the heat transfer and shear flow problems are now entirely similar in this new framework, and the criticisms of the shear flow results contained in Section IV. 1 are equally applicable to the heat transfer problem.

Of course this simple device is unacceptable in more general flow problems involving streamline curvature, because $p_{rr} \neq p_{\theta\theta} \neq 0$, even when $M^2 << 1$. For example, Goldberg's[11] solution of the linearized Grad equations for "slow" flow over a sphere exhibits the same contradictions in the heat transfer rate in the limit $Re/M \longrightarrow 0$. Here p_{rr} and $p_{\theta\theta}$ do not vanish identically even in the classical Navier-Stokes limit ($Re/M >> 1$), which corresponds to Stokes flow over a sphere. When dissipation is considered (M^2 arbitrary) these normal stresses do not vanish identically even in the simplest geometry of plane Couette flow, and cross-couplings between these stresses and the heat flux are inevitable if the unmodified Grad f is employed. Similar cross-couplings are observed in the problem of the steady, plane shock wave[12], and these cross-couplings are probably responsible for the difficulties that have been encountered in applying Grad's method to this problem.

These remarks are applicable to any f that is a simple extension of the Chapman-Enskog polynomial. Perhaps these difficulties can be avoided by utilizing the two-sided polynomial distribution of the form

$$f = f_o \left[1 + a_o^{\pm} \; c_x + a_1^{\pm} \; c_x c_y + \dots \right],$$

employed by Gross[10], Jackson, and Ziering for plane,parallel geometry. However, to the author's present knowledge this type of f has been applied only to the case of linearized flows ($M^2 << 1$). It is not clear that such a velocity distribution function can describe the situation for non-linear, highly rarefied gas flows, where two distinctly different Maxwellian streams are usually involved. In this connection we remark that the weighting function (f) introduced in Reference 1, which utilizes two half-range Maxwellians expressed in terms of a certain number of parametric functions, leads to physically consistent results over the whole range of gas density, not only for linearized, cylindrical Couette flow*, but also for non-linear plane Couette flow[13]. In addition, the moment equations derived for the steady plane shock wave do not exhibit any singularities within the shock wave. Of course no integral method is unique, but it appears that the choice of the weighting function f to be employed in Maxwell's moment method is dictated by the requirements imposed in the limiting case of highly rarefied gas flows, as well as in the classical Navier-Stokes regime.

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* Report in preparation.

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APPENDIX I

GRAD'S THIRTEEN MOMENT EQUATIONS IN TWO-DIMENSIONAL CYLINDRICAL COORDINATES

In calculating flow problems one is often mainly interested in certain lower moments of the velocity distribution function rather than the function itself. Therefore, it is natural that one takes the Maxwell integral transport equation as the starting point for applying approximate methods.

The equation is given as

$$\frac{\partial}{\partial t}\int f Q \, d\vec{\xi} + \nabla_R \cdot \left[\int f \vec{\xi} Q \, d\vec{\xi}\right] = \int f \left(\frac{\vec{F}}{m} \cdot \overrightarrow{\nabla_\xi Q}\right) d\vec{\xi} + \Delta Q \qquad \text{(I. 1)}$$

where

- f is the velocity distribution function
- Q is any function of the velocity components of a particle (moment, energy, etc.)
- $\vec{\xi}$ and $\vec{R}$ are independent variables
- $\vec{F}$ is the external force vector

and

$$\Delta Q = \iiiint (Q - Q') \, f \, f_1 \, V \, d\vec{\xi} \, d\vec{\xi}_1 \, b \, d b \, d h \, d\varepsilon \qquad \text{(I. 2)}$$

is the collision integral in which $Q' - Q$ represents the change in Q experienced in a collision.

In Grad's thirteen-moment approximation the distribution function is a linear function of the stresses and heat fluxes, which are now regarded as separate dependent variables not explicitly related to ρ, $\vec{u}$, T, and their derivatives. They are, however, related to the second and third moments of the velocity distribution function f. Thus, in a rectangular Cartesian coordinate system

$$f = f_o \left[1 + \frac{p_{ij}}{2pRT} c_i c_j - \frac{q_i c_i}{pRT}\left(1 - \frac{c^2}{5RT}\right)\right] \tag{I. 3}$$

where f_o is the local Maxwellian.

By substituting this expression for f (I. 3) into the equation (I. 1), and by taking Q to be equal successively to m , $m \xi_i$, $m(\xi^2/2)$, $m \xi_i \xi_j$ and $m \xi_i(\xi^2/2)$, the thirteen partial differential equations in a rectangular coordinate system (including the conservation relations) are obtained for the thirteen independent moments ρ, $\vec{u}$, T , p_{ij} and q_i . The equations are

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_r}(\rho u_r) = 0 \tag{I. 4}$$

Conservation of Momentum

$$\frac{\partial u_i}{\partial t} + u_r \frac{\partial u_i}{\partial x_r} + \frac{1}{\rho} \frac{\partial P_{i\alpha}}{\partial x_\alpha} = 0 \tag{I. 5}$$

Conservation of Energy

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x_\alpha}(u_\alpha p) + \frac{2}{3} P_{i\alpha} \frac{\partial u_i}{\partial x_\alpha} + \frac{2}{3} \frac{\partial q_r}{\partial x_r} = 0 \tag{I. 6}$$

Stresses

$$\frac{\partial p_{ij}}{\partial t}+\frac{\partial}{\partial x_\alpha}(u_\alpha p_{ij})+\frac{2}{5}\left(\frac{\partial q_i}{\partial x_j}+\frac{\partial q_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial q_\alpha}{\partial x_\alpha}\right)$$

$$+p_{i\alpha}\frac{\partial u_j}{\partial x_\alpha}+p_{j\alpha}\frac{\partial u_i}{\partial x_\alpha}-\frac{2}{3}\delta_{ij}p_{rs}\frac{\partial u_r}{\partial x_s} \tag{I. 7}$$

$$+p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial u_\alpha}{\partial x_\alpha}\right)=-\frac{p}{\mu}p_{ij}$$

Heat Fluxes

$$\frac{\partial q_i}{\partial t}+\frac{\partial}{\partial x_\alpha}(u_\alpha q_i)+\frac{7}{5}q_r\frac{\partial u_i}{\partial x_r}+\frac{2}{5}q_r\frac{\partial u_r}{\partial x_i}+\frac{2}{5}q_i\frac{\partial u_\alpha}{\partial x_\alpha}$$

$$+RT\frac{\partial p_{i\alpha}}{\partial x_\alpha}+\frac{7}{2}p_{i\alpha}\frac{\partial RT}{\partial x_\alpha}-\frac{p_{i\alpha}}{\rho}\frac{\partial P_{rs}}{\partial x_s}+\frac{5}{2}p\frac{\partial RT}{\partial x_i}=-\frac{2}{3}\frac{p}{\mu}q_i \tag{I. 8}$$

where the results of stresses are already utilized in the heat flux equations.

Given below also as a reference is the list of all moments involved in Grad's approximation.

$$\rho(\vec{R},t)=\int m f(\vec{\xi},\vec{R},t)\,d\vec{\xi} \tag{I. 9}$$

$$\rho\vec{u}(\vec{R},t)=\int m\vec{\xi} f\,d\vec{\xi} \tag{I. 10}$$

$$P_{ij}=\int m c_i c_j f\,d\vec{\xi} \tag{I. 11}$$

$$q_{ijk}(\vec{R},t)=\frac{1}{2}\int m c_i c_j c_k f\,d\vec{\xi} \tag{I. 12}$$

where $\vec{c}=\vec{\xi}-\vec{u}(\vec{R},t)$ is the intrinsic or relative velocity.

By contraction, the following tensors are produced.

$$P_{ii} = 3p \tag{I. 13}$$

$$q_{ijj} = q_i \tag{I. 14}$$

$$p_{ij} = P_{ij} - p\,\delta_{ij} \tag{I. 15}$$

$$p_{ii} = 0 \tag{I. 16}$$

Grad's Equations in Two-Dimensional Cylindrical Coordinate System

In two-dimensional problems, all quantities are independent of z, hence we may set $w = p_{xz} = p_{yz} = q_z = 0$ a priori, and the number of partial differential equations is reduced to nine.

To write these equations in cylindrical coordinates, one applies the following transformations

$$r = (x^2 + y^2)^{\frac{1}{2}} \qquad \theta = \tan^{-1}\frac{y}{x} \tag{I. 17}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_r \\ u_\theta \end{pmatrix} \tag{I. 18}$$

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} q_r \\ q_\theta \end{pmatrix} \tag{I. 19}$$

and

$$p_{ij} = l_{i\alpha}\, l_{j\beta}\, p_{\alpha\beta} \tag{I. 20}$$

where i , j are related to x , y and α , β are related to r , θ coordinates.

$l_{i\alpha}$ and $l_{j\beta}$ are direction cosines between the two coordinate systems

$$\begin{pmatrix} l_{xr} & l_{yr} \\ l_{x\theta} & l_{y\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

The nine moment equations become (without external force)

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right) + u_r\frac{\partial \rho}{\partial r} + \frac{u_\theta}{r}\frac{\partial \rho}{\partial \theta} = 0 \tag{I. 21}$$

Conservation of Momentum

r-component

$$\frac{\partial u_r}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\left(\frac{\partial p_{rr}}{\partial r} + \frac{1}{r}\frac{\partial p_{r\theta}}{\partial \theta} + \frac{p_{rr} - p_{\theta\theta}}{r}\right) + \left(u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r}\right) = 0 \tag{I. 22}$$

θ-component

$$\frac{\partial u_\theta}{\partial t} + \frac{1}{\rho}\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{1}{\rho}\left(\frac{\partial p_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial p_{\theta\theta}}{\partial \theta} + \frac{2}{r}p_{r\theta}\right) + \left(u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r}\right) = 0 \tag{I. 23}$$

Conservation of Energy

$$\frac{\partial p}{\partial t} + \frac{5}{3}p\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right) + \frac{2}{3}\left(\frac{\partial q_r}{\partial r} + \frac{q_r}{r} + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta}\right) + \left(u_r\frac{\partial p}{\partial r} + \frac{u_\theta}{r}\frac{\partial p}{\partial \theta}\right)$$
$$+ \frac{2}{3}\left(p_{rr}\frac{\partial u_r}{\partial r} + p_{r\theta}\frac{\partial u_\theta}{\partial r} + \frac{p_{r\theta}}{r}\frac{\partial u_r}{\partial \theta} + \frac{p_{\theta\theta}}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{p_{\theta\theta}}{r}u_r - p_{r\theta}\frac{u_\theta}{r}\right) = 0 \qquad \text{(I. 24)}$$

Stresses

"p_{rr}"

$$\frac{\partial p_{rr}}{\partial t} + \frac{2}{3}p\left(2\frac{\partial u_r}{\partial r} - \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r}\right) + \frac{4}{15}\left(2\frac{\partial q_r}{\partial r} - \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} - \frac{q_r}{r}\right) + u_r\frac{\partial p_{rr}}{\partial r} + \frac{u_\theta}{r}\frac{\partial p_{rr}}{\partial \theta}$$
$$-\frac{2}{r}u_\theta p_{r\theta} + \frac{7}{3}p_{rr}\frac{\partial u_r}{\partial r} - \frac{2}{3}p_{r\theta}\frac{\partial u_\theta}{\partial r} + \frac{4}{3}p_{r\theta}\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{4}{3}\frac{p_{r\theta}u_\theta}{r} - \frac{2}{3}\frac{p_{\theta\theta}}{r}\frac{\partial u_\theta}{\partial \theta} \qquad \text{(I. 25)}$$
$$-\frac{2}{3}\frac{p_{\theta\theta}}{r}u_r + \frac{p_{rr}}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{p_{rr}u_r}{r} = -\frac{p}{\mu}p_{rr}$$

"$p_{r\theta}$"

$$\frac{\partial p_{r\theta}}{\partial t} + p\left(\frac{\partial u_\theta}{\partial r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r}\right) + \frac{2}{5}\left(\frac{\partial q_\theta}{\partial r} + \frac{1}{r}\frac{\partial q_r}{\partial \theta} - \frac{q_\theta}{r}\right) + u_r\frac{\partial p_{r\theta}}{\partial r} + \frac{u_\theta}{r}\frac{\partial p_{r\theta}}{\partial \theta}$$
$$+ \frac{u_\theta p_{rr}}{r} - \frac{u_\theta p_{\theta\theta}}{r} + p_{rr}\frac{\partial u_\theta}{\partial r} + 2\frac{p_{r\theta}}{r}\frac{\partial u_\theta}{\partial \theta} + 2p_{r\theta}\frac{u_r}{r} + 2p_{r\theta}\frac{\partial u_r}{\partial r} + \frac{p_{\theta\theta}}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) \qquad \text{(I. 26)}$$
$$= -\frac{p}{\mu}p_{r\theta}$$

"$p_{\theta\theta}$"

$$\frac{\partial p_{\theta\theta}}{\partial t} - \frac{2}{3}p\left(\frac{\partial u_r}{\partial r} - \frac{2}{r}\frac{\partial u_\theta}{\partial \theta} - 2\frac{u_r}{r}\right) - \frac{4}{15}\left(\frac{\partial q_r}{\partial r} - \frac{2}{r}\frac{\partial q_\theta}{\partial \theta} - 2\frac{q_r}{r}\right) + u_r\frac{\partial p_{\theta\theta}}{\partial r} + \frac{u_\theta}{r}\frac{\partial p_{\theta\theta}}{\partial \theta}$$

$$+\frac{2}{r}u_\theta p_{r\theta} + \frac{4}{3}p_{r\theta}\frac{\partial u_\theta}{\partial r} - \frac{2}{3}\left(p_{rr}\frac{\partial u_r}{\partial r} + \frac{p_{r\theta}}{r}\frac{\partial u_r}{\partial \theta} - p_{r\theta}\frac{u_\theta}{r}\right) + \frac{7}{3}p_{\theta\theta}\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right) \quad \text{(I. 27)}$$

$$+ p_{\theta\theta}\frac{\partial u_r}{\partial r} = -\frac{p}{\mu}p_{\theta\theta}$$

Heat Fluxes

"q_r"

$$\frac{\partial q_r}{\partial t} + \frac{5}{2}p\frac{\partial RT}{\partial r} + RT\left(\frac{\partial p_{rr}}{\partial r} + \frac{1}{r}\frac{\partial p_{r\theta}}{\partial \theta} + \frac{p_{rr} - p_{\theta\theta}}{r}\right) + q_r\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right)$$

$$+u_r\frac{\partial q_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial q_r}{\partial \theta} - \frac{u_\theta q_\theta}{r} + \frac{11}{5}q_r\frac{\partial u_r}{\partial r} + \frac{2}{5}\left(q_r\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + q_r\frac{u_r}{r} + q_\theta\frac{\partial u_\theta}{\partial r}\right)$$

$$+\frac{7}{5}q_\theta\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r}\right) - \frac{1}{\rho}\left(p_{rr}\frac{\partial p}{\partial r} + \frac{p_{r\theta}}{r}\frac{\partial p}{\partial \theta}\right) + \frac{7}{2}\left(p_{rr}\frac{\partial RT}{\partial r} + \frac{p_{r\theta}}{r}\frac{\partial RT}{\partial \theta}\right) \quad \text{(I. 28)}$$

$$-\frac{p_{rr}}{\rho}\left(\frac{\partial p_{rr}}{\partial r} + \frac{1}{r}\frac{\partial p_{r\theta}}{\partial \theta} + \frac{p_{rr} - p_{\theta\theta}}{r}\right) - \frac{p_{r\theta}}{\rho}\left(\frac{\partial p_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial p_{\theta\theta}}{\partial \theta} + 2\frac{p_{r\theta}}{r}\right) = -\frac{2}{3}\frac{p}{\mu}q_r$$

"q_θ"

$$\frac{\partial q_\theta}{\partial t} + \frac{5}{2}p\frac{1}{r}\frac{\partial RT}{\partial \theta} + RT\left(\frac{\partial p_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial p_{\theta\theta}}{\partial \theta} + 2\frac{p_{r\theta}}{r}\right) + q_\theta\left(\frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right)$$

$$+u_r\frac{\partial q_\theta}{\partial r} + \frac{u_\theta}{r}\left(\frac{\partial q_\theta}{\partial \theta} + q_r\right) + \frac{11}{5}\left(q_\theta\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + q_\theta\frac{u_r}{r}\right) + \frac{7}{5}q_r\frac{\partial u_\theta}{\partial r}$$

$$+\frac{2}{5}\left(\frac{q_r}{r}\frac{\partial u_r}{\partial \theta} - q_r\frac{u_\theta}{r} + q_\theta\frac{\partial u_r}{\partial r}\right) - \frac{1}{\rho}\left(p_{r\theta}\frac{\partial p}{\partial r} + \frac{p_{\theta\theta}}{r}\frac{\partial p}{\partial \theta}\right) + \frac{7}{2}\left(p_{r\theta}\frac{\partial RT}{\partial r} + \frac{p_{\theta\theta}}{r}\frac{\partial RT}{\partial \theta}\right) \quad \text{(I. 29)}$$

$$-\frac{p_{r\theta}}{\rho}\left(\frac{\partial p_{rr}}{\partial r} + \frac{1}{r}\frac{\partial p_{r\theta}}{\partial \theta} + \frac{p_{rr} - p_{\theta\theta}}{r}\right) - \frac{p_{\theta\theta}}{\rho}\left(\frac{\partial p_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial p_{\theta\theta}}{\partial \theta} + 2\frac{p_{r\theta}}{r}\right) = -\frac{2}{3}\frac{p}{\mu}q_\theta$$

APPENDIX II

CALCULATION OF HEAT LOSS FROM VERY THIN HEATED WIRES IN A RAREFIED GAS BASED ON A TRUNCATED FORM OF GRAD'S THIRTEEN MOMENT METHOD

H. J. Bomelburg[14] has performed a series of experiments with fine heated wires of different diameters fastened inside a bell jar to study quantitatively the behavior of heat conductivity in rarefied gases. The temperature of the wire was kept constant and the heat loss at various pressures as measured. The heat loss at normal density ($Kn^* = \infty$) is defined as Q_∞, and the heat loss at some lower pressure is called Q. The quantity Q/Q_∞ is then plotted against Kn on a logarithmic scale to show that the heat conductivity is dependent on pressure as the mean free path gets to be large compared with the container. The wires used are of aspect ratio well above 1000 and the temperature difference $(T_w - T_b)/T_w$ is approximately 1/10. [Here T_w is the temperature of the wire, and T_b is the temperature of the gas at the wall of the bell jar. In these experiments T_w was about $60^\circ C$ and T_b was about room temperature $25^\circ C$.]

Bomelburg's experiment is very closely related to the linearized cylindrical Couette flow, since the geometry is the same and the temperature difference small. On the other hand, since the bell jar as well as the hot wire are fixed, there is no mean fluid motion, hence the problem

---

* Bomelburg defined Kn as d/λ, d being the diameter of the wire and λ the mean free path. His definition is just the reciprocal of the Kn commonly used.

reduces to a pure heat conduction. We hereby propose to treat the case by means of a truncated form of Grad's moment method. [See Section IV, Discussion and Conclusion.] (The problem can be idealized as two-dimensional because of the high aspect ratio and the boundary condition can be linearized because of small temperature difference.) First of all, let us introduce the following symbols:

- a — radius of wire
- b — radius of bell jar
- $Re/M = \dfrac{\rho_w a (\gamma R T_w)^{\frac{1}{2}}}{\mu_w}$
- ρ_w, μ_w, p_w — density, coefficient of viscosity, and pressure of gas at the wire surface.

We say a priori that $p_{rr} = p_{\theta\theta} = 0$ and $p = p_w$ = constant throughout the field. Furthermore, the two stress equations (p_{rr} and $p_{\theta\theta}$) are not used. Symmetry requires that all tangential quantities must vanish. The heat loss Q is given by the energy equation as

$$Q = c/r \quad .$$

The heat flux equation becomes

$$Q = -\ 15/4 \quad R \mu_w (dT/dr)$$

with the boundary conditions

at r = a

$$\left(\frac{2\pi}{RT_w}\right)^{\frac{1}{2}} \frac{Q(a)}{p_w} + \frac{4(1-\alpha)}{(1+\alpha)}\left[\frac{T_b - T_w}{T_w} + \frac{4}{15 R \mu_w}\frac{c}{T_w} \ln \frac{b}{a}\right] = 0$$

at r = b

$$T = T_b \quad .$$

Solving for c , we obtain

$$\frac{Q}{Q_\infty} = \frac{\frac{4(1-\alpha)}{(1+\alpha)}\frac{4}{15}\frac{Re}{M}\ln\frac{b}{a}}{\left[\sqrt{2\pi\gamma} + \frac{4(1-\alpha)}{(1+\alpha)}\frac{4}{15}\frac{Re}{M}\ln\frac{b}{a}\right]}$$

Re/M is proportional to Kn or

$$Re/M = \sqrt{5\pi/6} \quad Kn \quad .$$

For a diffusively reflected surface $\alpha = 0$,

$$Q/Q_\infty = \frac{\frac{16}{15}\sqrt{\frac{5\pi}{6}}\ln\frac{b}{a}Kn}{\sqrt{2\pi\gamma} + \frac{16}{15}\sqrt{\frac{5\pi}{6}}\ln\frac{b}{a}Kn} \quad .$$

Q/Q_∞ vs. Kn is plotted in Figure 7 on log-log scale. For small values of Kn, the two sided solid angle effect becomes more and more important and the curve deviates away from experimental results; however, it shows qualitatively the correct trend. The problem is now being studied at this laboratory by Mrs. Y. L. Wu, utilizing the two-sided Maxwellian introduced in Reference 1.

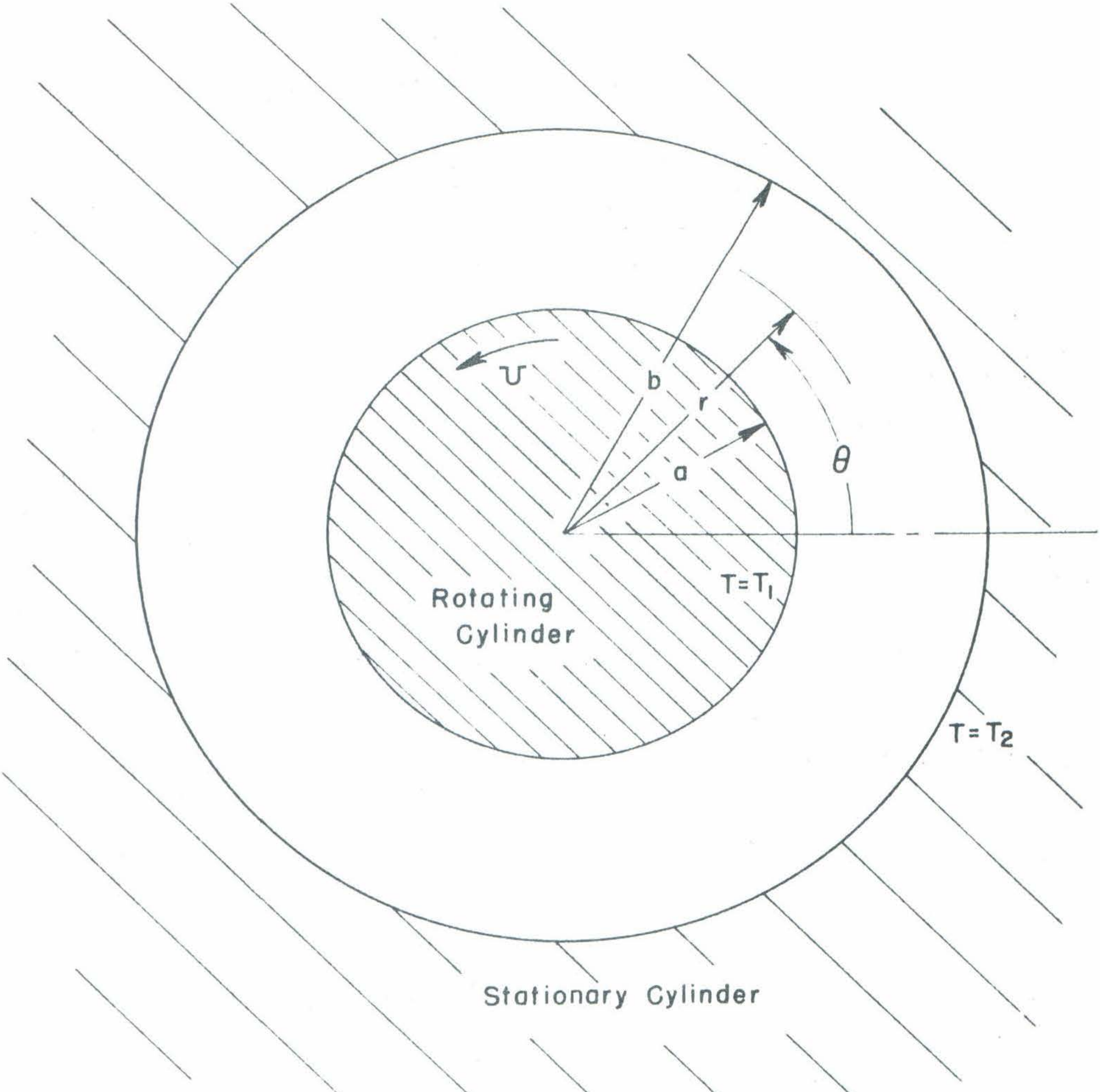

FIG. I - SCHEMATIC PICTURE OF THE PROBLEM

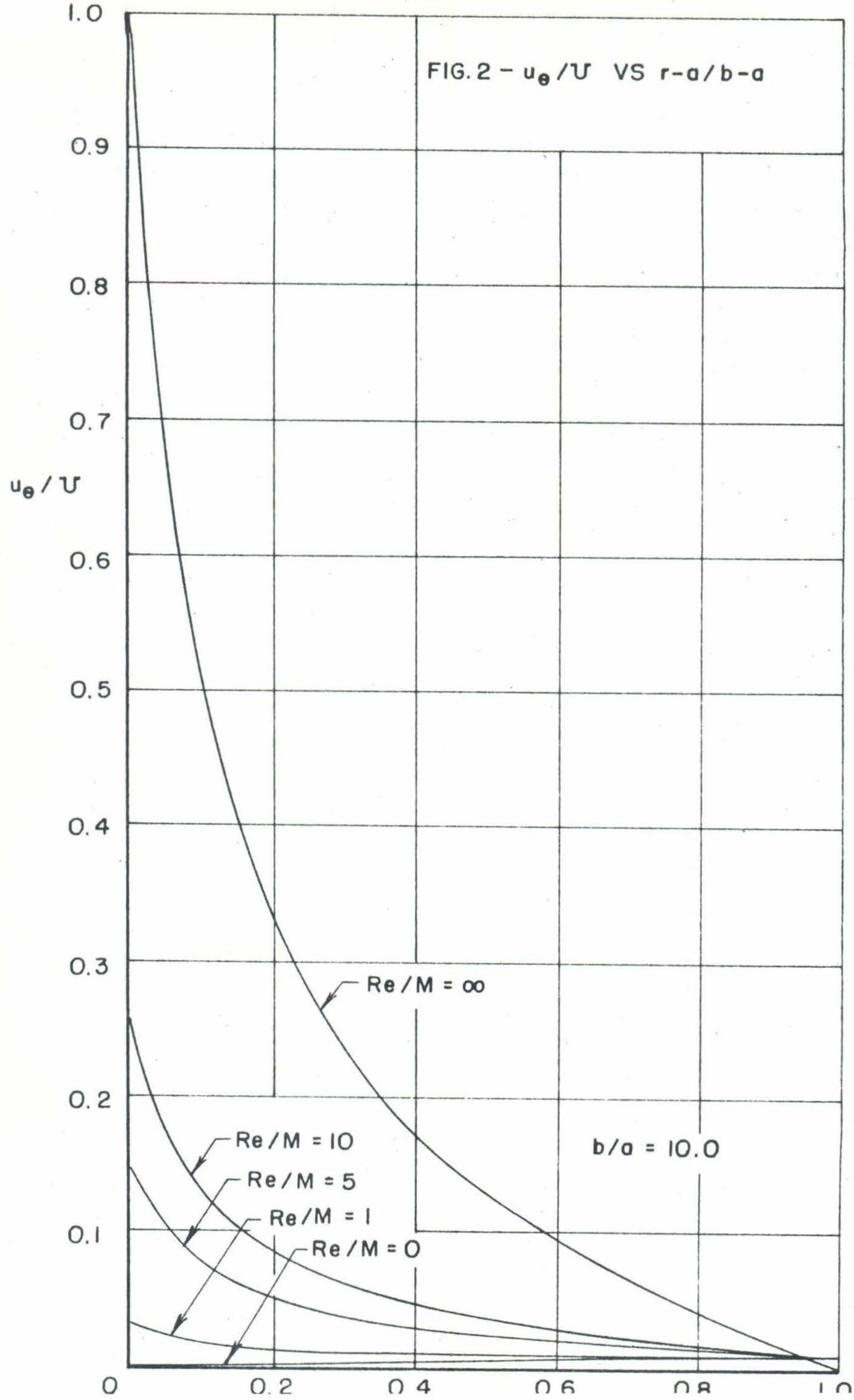
FIG. 2 – u_θ/U VS r-a/b-a
u_θ/U
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1.0
Re/M = ∞
Re/M = 10
Re/M = 5
Re/M = 1
Re/M = 0
b/a = 10.0

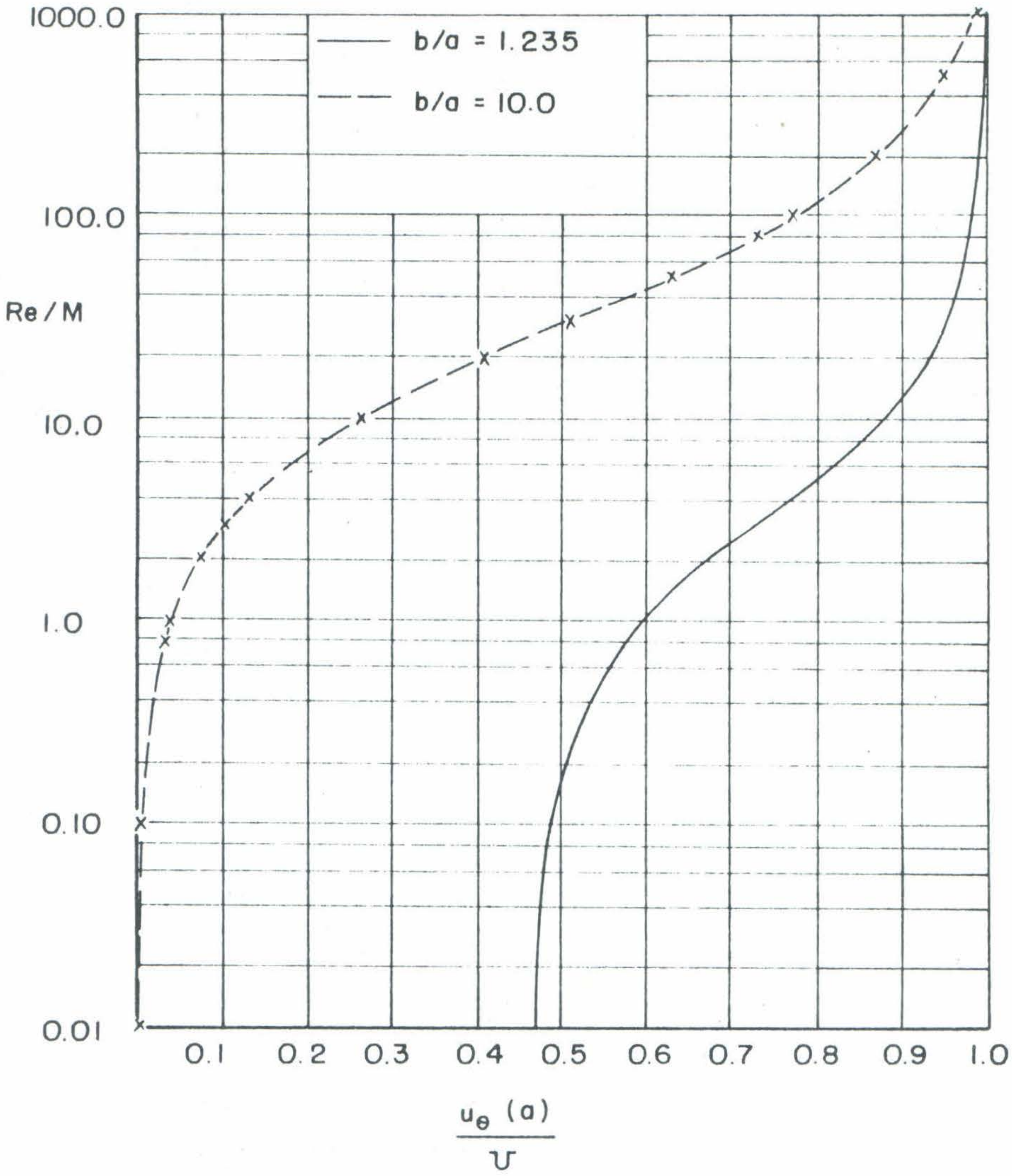

FIG. 3 - $u_\theta(a)/U$ VS Re/M

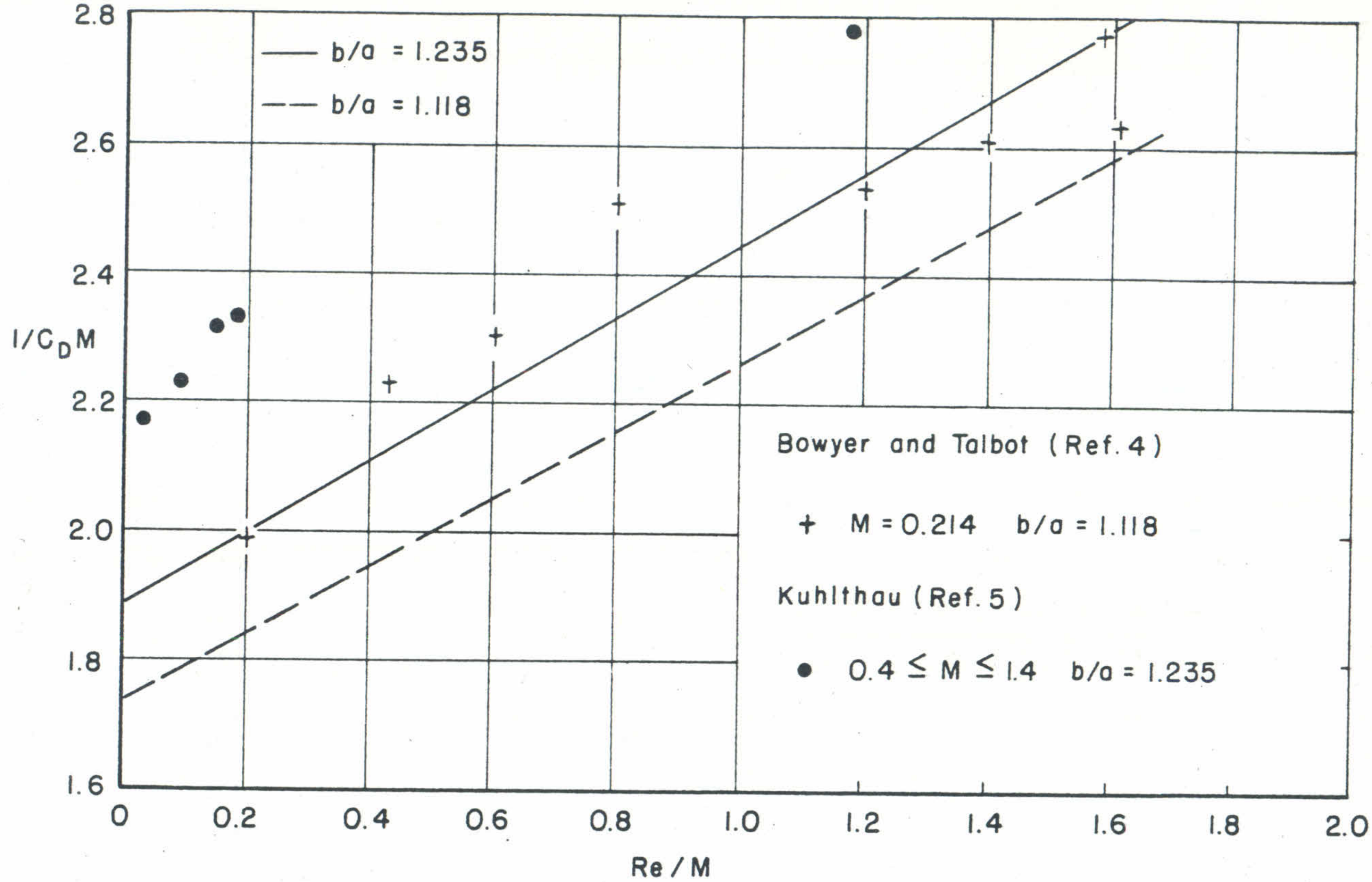

FIG. 4 – $1/C_D M$ VS Re/M

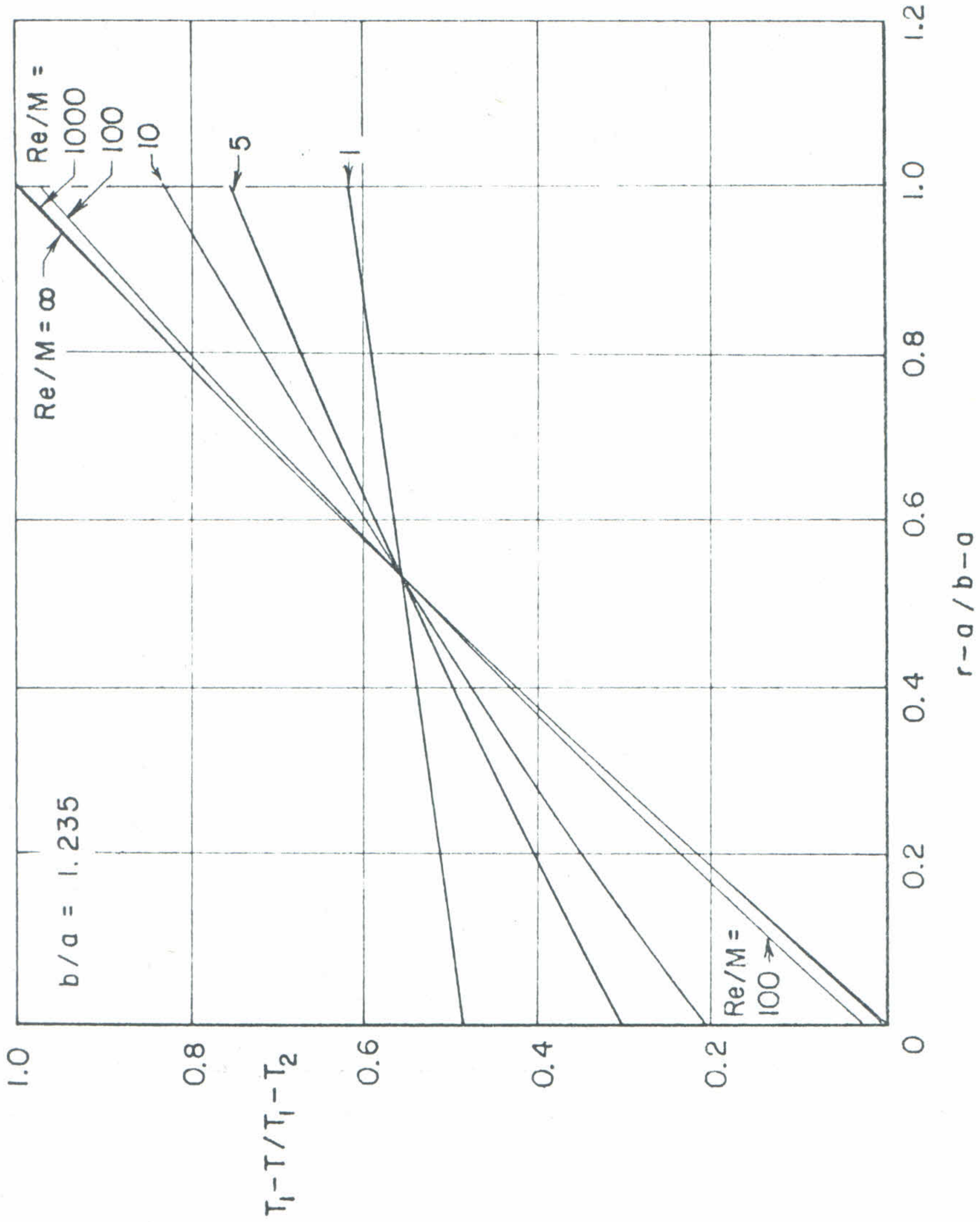

FIG. 5 — $T_1 - T / T_1 - T_2$ VS. $r - a / b - a$

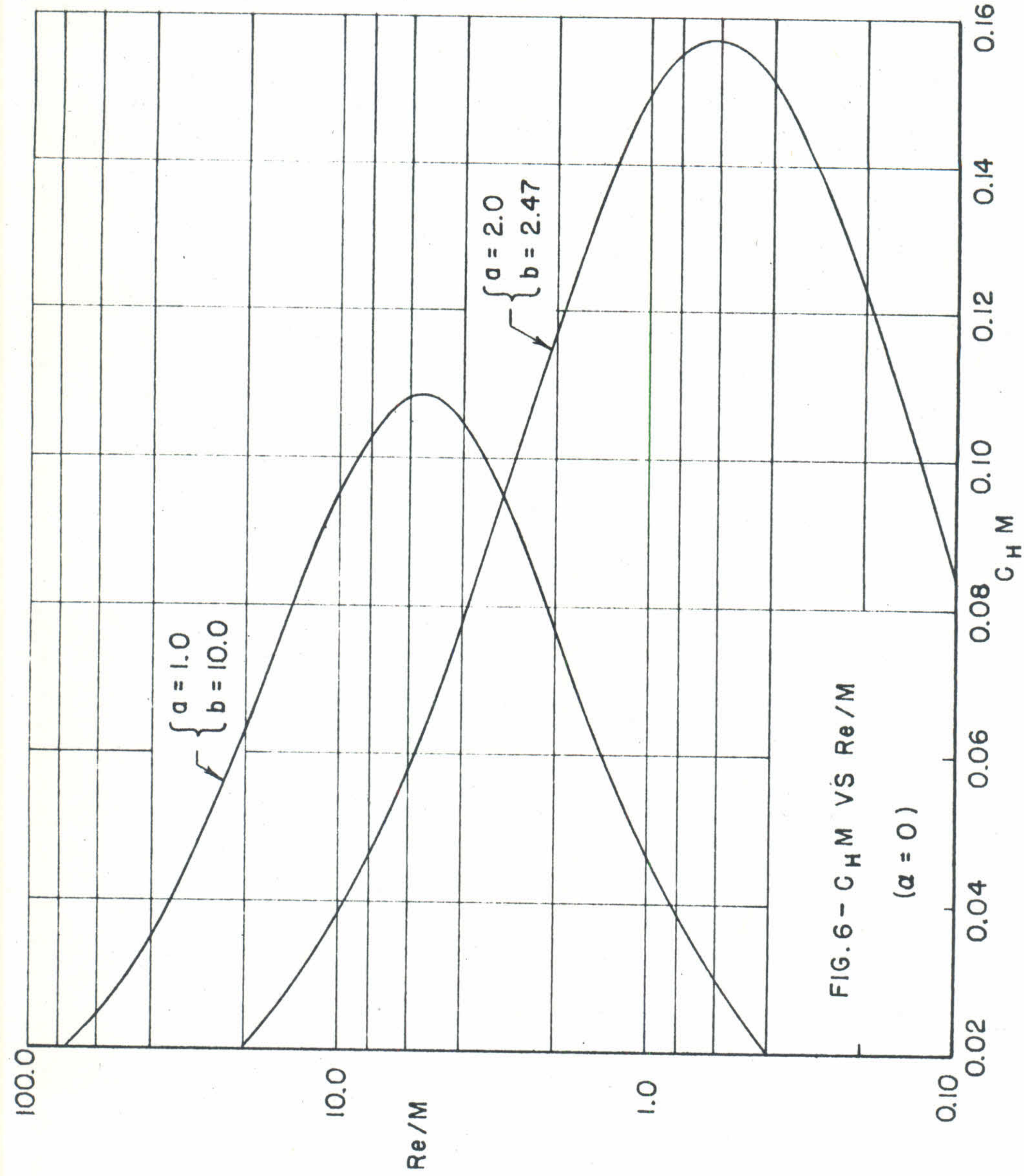

FIG. 6 - $C_H M$ VS Re/M

($\alpha = 0$)

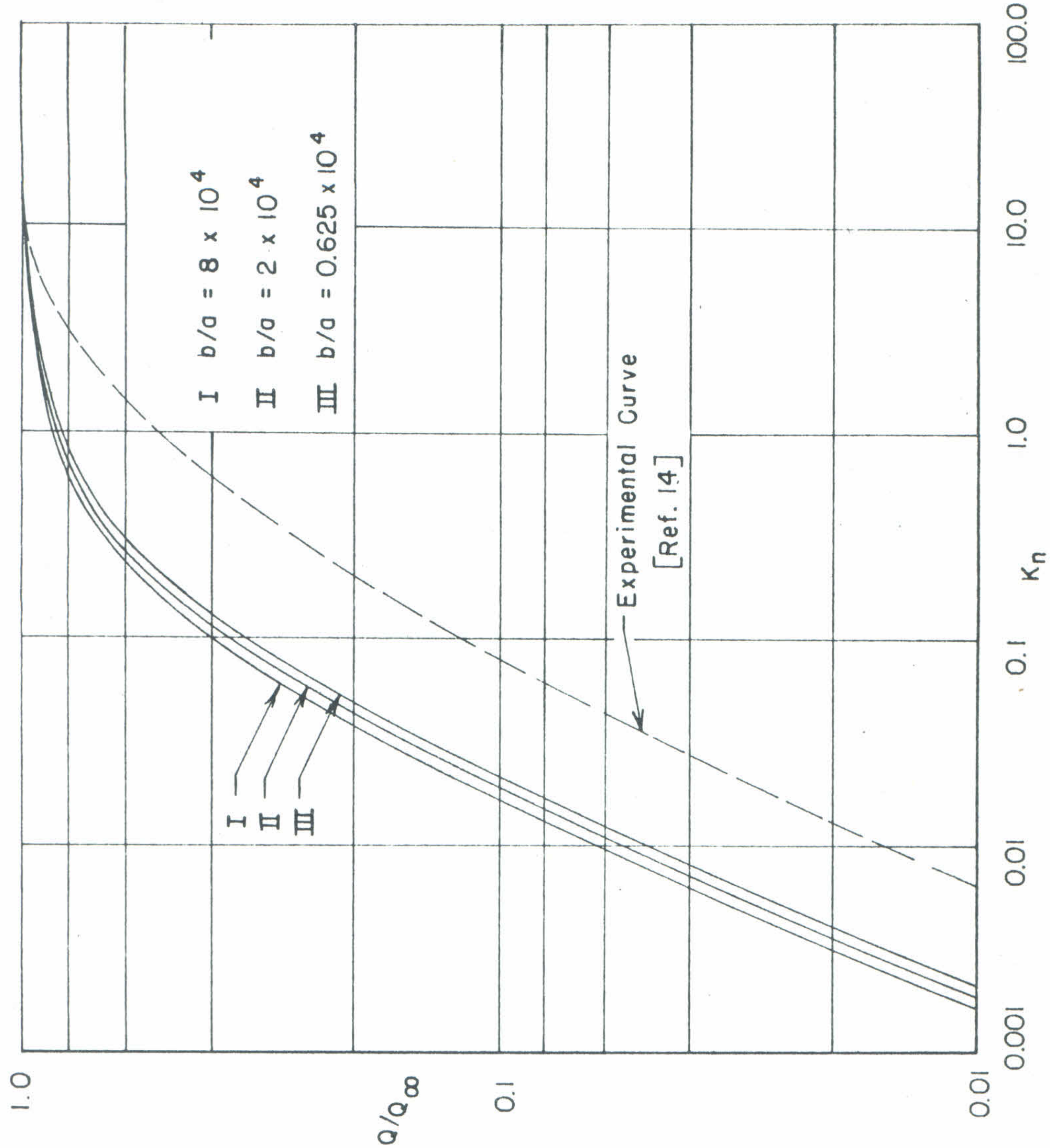

FIG. 7 - Q / Q_∞ VS. K_n

1 February 1960

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